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ANALYSIS OF THE VIERENDEEL GIRDER BY BALANCING THE PANEL MOMENTS

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ANALYSIS OF THE VIERENDEEL GIRDER BY BALANCING THE PANEL MOMENTS

A. T. Diwan, A.M. ASCE[1]

SYNOPSIS

A special type of panel displacement which produces moments in the chords of the distorted panel only and no else where in the Vierendeel is developed. This enables the analysis to be done by successive operations of joint rotations and special panel displacements, by which the moments in the joints and in the panels are successively balanced.

INTRODUCTION

The Vierendeel girder is usually regarded as one of the highly redundant constructions, for which the analysis by the classical methods necessitates the solution of 3n equations of continuity where n is the number of panels in the girder. This in fact limits the practical possibility of using the classical methods successfully for the analysis of the Vierendeel girder, and the designer readily welcomes any other method of analysis in which these simultaneous equations can be avoided. In a previous work the author developed an exact solution for the Vierendeel and other similar systems, of successive panels, using a new concept of "Equivalent Elastic Panels."[2] In that method the Vierendeel is reduced to an equivalent system of "Virtual" panels and the analysis is directly made without the use of simultaneous equations.

In this paper, a relaxation method for the analysis of the Vierendeel girder using a special type of panel displacement is presented.

The difficulty in using the moment distribution method originally developed by Prof. H. Cross for the analysis of the Vierendeel has so far been in the fact the joints of the girder undergo certain displacements as well as rotations before the final loaded position is taken.

To allow for the joint translations in the case of a Vierendeel with parallel chords is in fact quite simple, since any relative vertical translation may be imposed on the vertical sides of any panel without producing moments except in the chords of that panel. In the general case of the Vierendeel with non-parallel chords such a relative upward or downward translation in the vertical side of any panel is accompanied by a horizontal displacement in the top joints which sets up heavy moments in the vertical members. If, however, this translation is associated with some joint rotations, a special type of panel displacement is obtained which leaves all panels to each side of the displaced

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2. "Mathematical Analysis of Continuous Arches and Frames Using the Principles of Un-loaded Elastic Models" by A. F. Diwan. Thesis presented for the degree of Ph.D. at the University of London 1948.

panel free from bending moments. This enables us to produce the required displacement in any panel without affecting the stresses or the equilibrium of other panels.

By successive operations of joint rotations and special panel displacements, all initial fixations on the Vierendeel are gradually relaxed to any required degree of accuracy, as will be fully explained hereafter. The application of the method to the following cases is illustrated;

1. B.M.D for any case of loading.
2. Direct evaluation of influence lines by the Muler Breslau's principle.
3. Effect of variable temperature changes, in the Vierendeel.

The method may be applied to other types of structures and to Vierendeel girders with variable cross sections.

Part 1. THEORY

1. The Special Panel Displacement:

Fig. (1-a) shows the dimensions and relative stiffness of members for panel ABCD in a Vierendeel girder.

Let joint D be displaced vertically upwards to D without allowing rotation in any joint of the panel. If the axial deformation of the members is disregarded; C will move to C_1, fig (1. b), such that:

$D D_1 = C C' = a.s$; $C' C_1 = a.b$ - fig (1.e)

"a" is any arbitrary constant.

The moments produced in the panel will be as shown in fig. (1.b).

If on the other hand, joints C and D are rotated through an angle $\phi = ab$; with no joint translation, the bending moment produced in the panel will be as shown in figure (1.c).

When the joint translation in fig. (1.b) is associated with the joint rotation in fig. (1.c) a type of displacement will result in panel ABCD, as shown in figure (1.d), in which bending moments will be set up in the chords AC and BD only while the verticals CD and AD remain free from bending moments. In fact, the final displacement of the vertical side CD may be regarded as a vertical upward translation $\delta = a.s$, plus a rotation of the whole member through an angle $\phi = \frac{ab}{h_1}$ about the new position D_1 to which joint D has moved. Such a displacement would produce no moments in CD which remains un-distorted. All other panels to the right of CD will also remain unstressed. They are simply pushed as a rigid body upwards through a distance $\delta = a.s$; then rotated about joint D_1 through an angle $\phi = \frac{ab}{h_1}$. This is shown in figure. (2.a).

Refering to figure (1.d), the bending moments produced in chords AB and BD by the special displacement imposed on panel ABCD are:

$$\left.\begin{aligned}
M^{ac} &= M_1 = 2k_1 a\left(3 + \frac{4b}{h_1}\right) \\
M^{ca} &= M_2 = 2k_1 a\left(3 + \frac{2b}{h_1}\right) \\
M^{bd} &= M_3 = 2k_2 a\left(3 + \frac{b}{h_1}\right) \\
M^{db} &= M_4 = 2k_2 a\left(3 + \frac{2b}{h_1}\right)
\end{aligned}\right\} \qquad (1)$$

Eqns (1) may be written in the form:

$$\left.\begin{aligned}
M_1 &= a_1 k_1 (2h_1 + h_2) \\
M_2 &= a_1 k_1 (h_1 + 2h_2) \\
M_3 &= a_1 k_2 (2h_1 + h_2) \\
M_4 &= a_1 k_2 (h_1 + 2h_2)
\end{aligned}\right\} \qquad (2) \qquad \longleftarrow \text{In which } a_1 = \frac{2a}{h_1}$$

From Equations, (2) it follows that:

$$M_1 : M_2 = M_3 : M_4 = (2h_1 + h_2) : (h_1 + 2h_2)$$

This means that;

1. **Points G_1 and G_2, where the bending moment is zero in chords AC and BD, will lie on a vertical line G_1G_2.**
2. **Line G_1G_2 will pass through the centre of the plane area ABCD. This easily fixes the ratio of the moments $M_1 : M_2$ or $M_3 : M_4$.**
3. **The moments in the top and bottom chords along any vertical section are directly proportional to the stiffness k_1 and k_2 of the chords. This again fixes the ratio $M_1 : M_3$ or $M_2 : M_4$.**

Since no bending moments are produced in the vertical members of the Vierendeel truss due to the displacement imposed on panel ABCD, no horizontal forces are set up in chords AC and BD. The external action on the panel necessary to produce this displacement will consist of the four moments M_1, M_2, M_3 and M_4 plus two vertical shearing forces $V = -\Sigma M + S$ as shown in figure (2.b.).

2. Condition for the Static Equilibrium in the Panel.

Consider figure (3-a) which shows an intermediate panel in a Vierendeel truss under any case of loading. Only the chords AC and BD of panel ABCD are shown, the rest of the Vierendeel including the vertical sides AB and CD being removed and substituted by its action on the chords at A,B,C and D. The moments and forces in figure (3) represent the action of the joints on the chords.

Taking moments at B say, we get for the equilibrium of the panel:

$$\Sigma M_c + Q^d \times s + P.a - H.b = o$$

or

$$\Sigma M_c + Q'.s - H.b = o \qquad (3)$$

in which:

$\Sigma M_c = M_1 + M_2 + M_3 + M_4$

= sum of the moments applied by the joints on the chords. Clockwise moments are positive.

Q' = modified shearing force in panel ABCD, obtained on the assumption that the intermediate loads in the panel, if any, are substituted by their simple action at the ends of the chord BD. This will not change the end reactions in the Vierendeel. Q' will then be equal to the sum of all forces to either side of any intermediate section in the panel, and will be constant throughout the panel. Q' is positive when it produces clockwise rotation.

H = horizontal force acting on the top joint of the shorter side of the panel. It is positive when it acts from left to right.
H equals the sum of the shearing forces in the vertical members to the right of CD in figure (3.a), or to the left of CD in figure 3.b, including CD itself. The shearing force in any vertical member equals $\frac{\Sigma M_v}{h}$ where:

ΣM_v = sum of the two moments applied by the top and bottom joints on the vertical member.

h = length of the vertical member.

Equation (3) applies also to panel ABCD in figure (3.b). When no intermediate loads act between the panel points, equation (3) reduces to:

$$\Sigma M_c = Q.s - H.b = o \qquad (3\text{-a})$$

3. Balancing Coefficients for the Moments in the Panel. (Fig. 2)

If the sum ΣMc of the moments in the chords of the panel does not satisfy eqns. (3) or (3.a), the panel is said to be unbalanced. The unbalanced moment in the panel will equal to M* where.

$$M^* = (\Sigma M_c + Q'.s - H.b.) \qquad (4)$$

To satisfy the panel equilibrium it becomes necessary to reduce the chord moments ΣM_c by an amount equal to M*. This is done by imposing a displacement similar to that in figure (2) on the panel. The magitude and direction of the displacement will depend on the value and sign of the unbalanced moment M*. If cs = distance of line G_1G_2 passing through the centre of the panel surface from the longer side AB, where s = length BD.

$$\therefore C = \frac{2h_1 + h_2}{3(h_1 + h_2)} \qquad (5)$$

and (1-c)s = distance of G_1G_2 from the shorter side CD.

n = some arbitrary constant.

then:

$$M_1 = nck_1$$
$$M_2 = n(1-c)k_1$$
$$M_3 = nck_2$$
$$M_4 = n(1-c)k_2$$
$$\Sigma M = n(k_1 + k_2)$$

or

$$n = \frac{\Sigma M}{k_1 + k_2}$$

For the displacement necessary to balance the panel;

$$\Sigma M = - M^*$$

Therefore:

$$M_1 = - M^* \cdot \frac{ck_1}{k_1 + k_2} = - \epsilon_1 M^*$$

$$M_2 = - M^* \cdot \frac{(1-c)k_1}{k_1 + k_2} = - \epsilon_2 M^*$$

$$M_3 = - M^* \cdot \frac{ck_2}{k_1 + k_2} = - \epsilon_3 M^* \qquad (6)$$

$$M_4 = - M^* \cdot \frac{(1-c)k_1}{(k_1 + k_2)} = - \epsilon_4 M^*$$

where:

$$\epsilon_1 = \frac{ck_1}{k_1 + k_2}$$

$$\epsilon_2 = \frac{(1-c)k_1}{k_1 + k_2}$$

$$\epsilon_3 = \frac{ck_2}{k_1 + k_2} \qquad (7)$$

$$\epsilon_4 = \frac{(1-c)k_2}{k_1 + k_2}$$

The terms $\epsilon_1, \epsilon_2, \epsilon_3$ and ϵ_4 are called the balancing coefficients for the panel. To balance the panel, the moment $- M^*$ is simply distributed on the chords according to these coefficients.

4. Proposed Method of Analysis:

In the final displaced position of the Vierendeel truss under any case of loading, the moments produced in the members must satisfy the equilibrium condition for all panels and joints.

For each panel: $\Sigma M_C + Q'.s - H.b = 0$ (a)

For each joint: $\Sigma M = 0$ (b)

Starting with condition (a), it is seen that the modified shear value Q' due to any particular case of loading, is known for all panels, since the Vierendeel is externally statically determinate. In the initial unloaded position of the Vierendeel, with no moments yet produced, $\Sigma M_C = H =$ zero for all panels. The unbalanced moment in each panel is therefor equal to $Q'.s$. This moment is distributed according to the balancing coefficients ϵ_1 to ϵ_4 between the chords of the panel. We notice that such balancing of any panel will not affect other panels in the Vierendeel, so that we can balance all the panels by balancing each individual panel separately. In doing so we have in fact imposed on each panel a special displacement of the type proposed in figures (i-d), or (2-a).

In the case when an influence line is to be drawn, or when some temperature

effects are to be studied or so, no reactions are set up at the supports, yet special moments must be applied in some manner as will be seen later. In such cases $Q' = o$ in all panels; and the unbalanced moment to start with in any panel equals the sum of the moments initially imposed in the chords of the panel, if any.

Now that condition (a) is satisfied for all panels, we proceed to condition (b) for the joints. Each joint is allowed to rotate while all other joints are kept fixed until the unbalanced joint moment is distributed among the near ends of the two chords and vertical framing into the joint according to their stiffness values k. Moments equal to half the distributed moments are carried over to the far ends of the members.

As a result of these joint rotations, moments are produced in the verticals, and this in turn gives rise to horizontal forces H in the panels originally taken equal to zero. In addition to this, the chord moments are changed by the distributed and carry-over moments developed by the joint rotations. The result in that condition (a) for the panel equilibrium is no longer maintained, and it needs readjustment. The new unbalanced moment in each panel will be.

$$M^* = \Sigma M'_c - H'.b$$

Where M'_c and H' are chord moments and the horizontal force produced by the previous step of joint rotations. This moment is distributed among the chords according to the chord balancing coefficients ϵ. This is followed by distributing the unbalanced moments at the joints. The process continues to the required degree of accuracy. It will be seen from the examples shown hereafter that the convergence of the method is remarkable.

Part II. APPLICATIONS

The method is now to be applied for the analysis of the Vierendeel truss shown in figure (4.a). The same truss was dealt with before by the author using the method of "Equivalent Elastic Panels."[3] It is interesting to compare both results and see the remarkable agreement.

The dimensions and relative stiffness values $k = \frac{EI}{L.EI_o}$ for all Vierendeel members are given in the figure. EI for the top chord members equals 3EI for the bottom chord and vertical members. EI_o is any arbitrary value taken equal to $\frac{1}{3}$ EI of the bottom chord or $\frac{1}{9}$ EI of the top chord. In fig. (4.b), half the values of the distribution factors due to joint rotations are shown. These in fact are the carry over factors for the moments developed at the far ends of the members. To save time and space the distributed moments at the joints produced by the joint rotations are not computed. Only the carry over moments are entered in the tabulated solutions given hereafter. In the final step however, the total unbalanced moment at each joint is distributed among the members framing into the joint.

The balancing coefficients for the panels are also shown in figure (4.c). The value c giving the position of the centre of area for each panel is first

3. "Mathematical Analysis of Continuous Arches and Frames Using the Principles of Un-loaded Elastic Models" by A. F. Diwan. Thesis presented for the degree of Ph.d. at the University of London. 1948.

determined from equation (5); then coefficients ϵ are computed from eqns. (7).

$$c = \frac{2h_1 + h2}{3(h_1 + h_2)}$$

For the first panel;

$h_1 = 1.2$; $h_2 = 2.28$, $k_1 = 2.82$, $k_2 = 1.0$

$$\therefore c = \frac{2(1.2) + 2.28}{3(1.2 + 2.28)} = 0.448.$$

From eqns. 7;

$$\epsilon_1 = 0.448 \left(\frac{2.82}{1 + 2.82}\right) = 0.330$$

$$\epsilon_2 = 0.552 \left(\frac{2.28}{1 + 2.28}\right) = 0.408$$

$$\epsilon_3 = 0.448 \left(\frac{1}{1 + 2.28}\right) = 0.117$$

$$\epsilon_4 = 0.552 \left(\frac{1}{1 + 2.28}\right) = 0.145$$

5. B.M.D. due to a load P = 7000 lb at L.

Table (1) shows the necessary computations for the analysis of the Vierendeel under a load P = 7000 lb. at joint L. In the initial position, with no moments yet produced, the unbalanced moment in each panel equals Q.S.

This equals - 2000 x 3 = - 6000 lb. ft in the five panels to the right and 5000 x 3 = 15000 lb. ft in the two panels to the left. The unbalanced moments are distributed and entered in line (1) in the table. Next the joints are balanced, and the carry over moments are entered in line (2). For joint B for example; the unbalanced moment is - (4950 + 5850) = - 10800 lb. ft. The carry over moment produced at end ϵ of chord BC will be equal to 0.206 (10800) = 2225 lb. ft. Needless to say that the distributed moment produced in end B of BC attached to joint B is twice the carry over moment; i.e. 4450 lb. ft.

In step (3) the panels are balanced again. Consider the first panel ABJK. The unbalanced moment due to the carry over moments only in step (2) will be:

$H' = (780 + 1440) \div 1.2 = 1850$ lb.

$\Sigma M'_c = (2160 + 1625 + 570 + 310) = 4665$ lb. ft.

$\therefore M^* = 4665 - 1850\ (1.08) = 2668$ lb. ft.

The total unbalanced moment in the panel is actually three times this moment, since the distributed moments not tabulated in step (2) are double the shown carry over moments. Therefore; the total unbalanced moment in the first panel will be:

$M^* = 3 \times 2668 = 8004$ lb. ft.

For the second panel BCKL; the carry over moments give:

$H' = 1850 + (745 + 1020) \div 2.28 = 2630$ lb.

$\Sigma M_c = (600 + 2225 + 177 + 570) = 3572$ lb. ft.

The unbalanced moment will be:

$M^* = 3\ [3572 - 0.72\ (2630)] = 5000$ lb. ft.

1	A	-6125	-4950	B	-5850	-5300	C	2290	2200	D	2250
2	+ 780	2160	1625	745	600	2225	177	- 965	625	- 232	- 970
3		-3270	-2640		-1950	-1770		1640	1570		2760
4	- 303	333	88	19	- 277	325	- 79	- 815	- 283	- 185	- 602
5		- 635	- 512		- 68	- 62		1370	1320		1970
6	46	150	160	50	- 156	155	- 30	- 482	- 160	- 94	- 426
7		- 240	- 195		204	185		1000	960		1266
	- 34	13	12	-12	- 180	13	- 46	- 336	- 175	- 71	- 283
		- 31	- 25		240	218		710	682		882
	4	- 7	14	- 4	- 118	- 7	- 28	- 226	- 121	- 48	- 199
		11	9		190	175		482	463		594
	- 4	- 18	- 2	- 8	- 83	- 19	- 21	- 150	- 84	- 32	- 133
		28	23		140	128		320	310		396
	- 1	- 14	- 2	- 5	- 55	- 14	- 14	- 121	- 56	- 21	- 90
		+ 22	18		96	87		237	227		288
	- 1	- 10	- 2	- 3	- 37	- 11	- 11	- 76	- 38	- 16	- 61
		16	13		+ 68	+ 62		195	187		192
18	3548	3782	5127	2408	5285	- 600	-207	-603	-6400	-1923	-6437
1		-2180	-1755		-2010	-1830		+770	+ 740		750
2	1440	570	310	1020	177	570	220	-256	177	- 290	- 256
3		-1160	- 935		- 670	- 610		544	525		920
4	78	15	- 122	152	- 79	15	- 96	-185	- 79	- 274	- 145
5		- 225	- 182		- 23	- 21		465	445		657
6	142	38	19	71	- 30	38	- 54	-104	- 30	- 144	- 112
7		- 86	- 70		70	61		335	316		422
	10	- 9	- 13	6	- 46	- 9	- 61	- 78	- 46	- 100	- 73
		- 11	- 9		82	75		240	230		294
	12	- 3	2	- 3	- 28	- 3	- 41	- 53	- 28	- 68	- 50
		4	3		66	60		162	153		198
	- 2	+ 6	- 2	- 8	- 21	- 6	- 29	- 35	- 21	- 45	- 34
		10	9		48	44		107	102		132
	- 1	- 4	0	- 7	- 14	- 4	- 19	- 23	- 14	- 36	- 22
		8	6		33	30		80	76		96
	- 1	- 3	0	- 5	- 11	- 3	- 13	- 17	- 11	- 23	- 17
		6	4		23	21		65	63		64
18	968	390	1100	1560	1190	- 102	-102	-102	-1530	-1384	-1530
	J			K			L			M	

Table (1). Moment Computations For a Single Load P = 7000 lbs. at L .

This is distributed between the chords according to the balancing coefficients; for example;

$M^{bc} = - 0.39\ (5000) = - 1950$ lb. ft.

$M^{cb} = - 0.354\ (5000) = - 1770$ lb. ft.

These moments are entered in line 3.

As we proceed towards the centre panel, "b" decreases and the term (b.H) diminishes. For the central panel DEMN the unbalanced moment equals three times the sum of the carry over moments.[4]

4. Notice that the sum of the shears in all verticals at this stage will not be zero. Some sort of support capable of giving a horizontal reaction only may

	E			F			G			H	
2250		2200	2290		2120	2340		1980	2450		1
- 970	- 232	- 950	- 965	- 250	- 890	- 930	- 300	- 650	- 865	- 310	2
2760		2160	2270		1630	1810		1060	1325		3
- 820	- 132	- 390	- 600	- 86	- 204	- 380	- 43	- 40	- 198	122	4
1970		1320	1375		760	835		297	365		5
- 484	- 102	- 270	- 425	- 69	- 138	- 263	- 51	- 77	- 134	- 23	6
1266		890	930		480	530		174	216		7
- 338	- 66	- 168	- 282	- 40	- 84	- 164	- 20	- 16	- 63	11	
882		600	622		284	312		86	106		
- 227	- 46	- 112	- 199	- 27	- 40	- 110	- 14	- 14	- 38	- 3	
594		402	424		180	197		50	61		
- 151	- 30	- 73	- 133	- 18	- 23	- 72	- 8	- 5	- 22	1	
396		273	285		117	129		29	35		
- 121	- 20	- 50	- 90	- 12	- 15	- 48	- 5	- 4	- 15	0	
288		184	190		78	86		17	21		
- 76	- 15	- 33	- 61	- 8	- 10	- 32	- 3	- 1	- 9	0	
192		128	133		53	59		11	14		
-5620	-1680	-5600	-4130	-1400	-4045	-2785	-1270	-2700	-1614	-1433	18

750		740	770		730	810		700	870		1
- 256	- 290	- 250	- 256	- 320	- 230	- 250	- 406	--124	- 230	- 575	2
920		720	760		560	620		375	465		3
- 185	- 180	- 86	- 145	- 132	- 33	- 86	- 93	+ 49	- 32	- 35	4
657		444	460		262	283		105	130		5
- 104	- 127	- 69	- 112	- 89	- 39	- 69	- 63	- 9	- 39	- 69	6
422		300	310		165	182		62	76		7
- 78	- 85	- 40	- 73	- 57	- 16	- 40	- 29	+ 5	- 16	- 14	
294		200	208		98	107		30	38		
- 53	- 60	- 27	- 50	- 38	- 11	- 27	- 18	- 1	- 11	- 13	
198		137	142		62	68		18	22		
- 35	- 40	- 18	- 34	- 25	- 6	- 17	- 11	- 0	- 6	- 5	
132		90	96		40	44		10	12		
- 23	- 27	- 12	- 22	- 17	- 4	- 12	- 7	- 0	- 4	- 3	
96		62	64		27	30		6	8		
- 17	- 18	- 8	- 17	- 11	- 3	- 8	- 4	- 0	- 2	- 1	
64		43	45		18	20		4	5		
-1435	-1310	-1435	-1025	-1025	-1025	- 685	- 890	- 685	- 163	- 407	18
	N			O			P			R	

Table (1) (continued.)

Next the unbalanced moments at the joints are distributed and the carry over moments entered in line 4, followed in line 5 by the moments necessary to re-establish the panel equilibrium.

Finally in line 18, we enter the total distributed moments at the joints. For joint B say, the sum of the moments in lines 1 to 17 is 12820 lb. ft., to be distributed to BA, BC and BK in the ratios - .40, - 0.412 and - .188 respectively.

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be assumed to exist at joint D to maintain the equilibrium of the top chord against the unbalanced horizontal force H. In any panel, H by definition is the horizontal force acting at the top joint of the shorter side. This will not be affected by the reaction of this support. As we proceed with the relaxation process, the reaction set up by the artificial support will gradually diminish, and should finally disappear at last.

The final moments are obtained by summation, and are given in table 2 for comparison with those previously obtained by the method of "Equivalent Elastic,"[5] which is refered to in the table by the exact method. The agreement of both results is indeed remarkable. The final B.M.D. is also shown in figure. (5).

Table 2 : Final Moments (lb.ft.) For P = 7000 lb at L;

	Relaxation method	Exact method		Relax. method	Exact method		Relaxation method	Exact method		Relax. method	Exact method.
AB	-3835	-3832	ED	1792	1846	JK	-2646	-2633	NM	1347	1377
BA	-1240	-1208	FF	531	572	KJ	-1545	-1532	NO	790	822
BC	-1950	-1980	FE	1635	1676	KL	-1245	-1231	ON	1120	1154
CB	-4210	-4340	FG	275	266	LK	-1674	-1695	OP	595	592
CD	+4470	4580	GF	1515	1500	LM	1868	+1911	PO	965	961
DC	1222	1266	GH	200	218	ML	1070	1077	PQ	555	537
DE	1400	1446	HG	1635	1633	MN	1294	1329	QP	1122	1125

6. Direct Evaluation of Influence Lines.

According to the Muler Bruslau's principle, the influence line for moment, thrust or shear at any section is the deflection line of the loaded chord produced by a unit relative rotation, axial translation or unit relative translation normal to the axis, imposed at that section. To impose such a relative displacement the Vierendeel is cut at that section until the required displacement is produced, after which full continuity is re-established at the section.

Since the Vierendeel is externally statically determinate, no reactions are produced at the supports due to the imposed relative displacement. All joints of the Vierendeel are initially kept fixed against rotation (and if possible against translation also) when the relative displacement is being imposed. The following two examples will illustrate the application of the method.

6-A. Influence line for the moment M^{cd}.

While all joints are kept fixed, a unit rotation is produced in end C of the chord CD. Figure (6) shows the Vierendeel with the deflected member CD only and the resulting bending moment diagram.

$$M^{cd} = 4k.\,\emptyset = 4k = 4\ (2.98) = 11.92 \qquad \text{“}\emptyset = \text{unity”}$$

$$M^{dc} = 2k\emptyset = 2k = 2\ (2.98) = 5.96$$

These moments are entered in table (3); line (1). No moments are yet produced any where in the Vierendeel since no reactions are set up at the end supports. Only panel CDLM is now unbalanced; and the unbalanced moment is:

$$M^{*} = 11.92 + 5.96 = 17.88$$

This moment is distributed according to the coefficients ϵ and the balancing moments are entered in line 2. In line 3 are the carry over moments and in line 4 the new balancing moments in the panels. Finally in line 15 we have the total distributed moment at each joint.

5. Thesis for Ph.D. London University. A. Diwan. 1948.

For convenience, all moments in table (3) are multiplied by 1000. The final moments are given in table (4) and may be compared with these obtained by the method of "Equivalent Elastic Panels". The influence line is shown in figure (7).

6-B. Influence Line for the Thrust in Chord LM.

We need to produce a unit relative axial translation in chord LM. Two possible ways for producing this displacement are shown in figure (8). In case (a) joints J, K and L are all translated to the left through the same distance without rotation.

In case (b), the 4 panels to the right are rotated as a rigid body through an angle = - ϕ about joint D and the two panels to the left are rotated through an angle - ϕ about joint C.

$$\phi = \frac{1}{(h_1 + h_2)} = 1 \div 6.36 = 0.157.$$

The same rotations ϕ and - ϕ are also imposed on the ends L and M respectively of chord LM which is supposed to be detached from the Vierendeel during the panel rotations about C and D. Of course it is supposed that after the relative translation is imposed, chord LM is reconnected rigidly to the Vierendeel. (by some rigid arms.) The bending moments produced in the chords CD and LM due to the translation in case (b) are as shown in the figure, in which;

$$M^{cd} = - M^{dc} = 2k_1\phi = 2(2.98)\ (0.157) = 0.940$$

$$M^{lm} = - M^{ml} = 2k_2\phi = 2(0.157) = 0.314$$

No moments are produced anywhere else in the Vierendeel.

It is not necessary that the imposed rotations at C and D should be equal and opposite, yet it is more convenient to do so since this would make panel CDLM balanced under the initial set of moments.

Type (b) for the imposed displacement is more favourable than type (a) and is used in the tabulated computations given hereafter (table 5).

The initial moments are entered in line 1. All panels, including CDLM, are balanced. So we proceed to evaluate the carry over moments in line 2, then the balancing moments in line 3, and finally the total distributed moments in line 20.

Table (6) shows the final moments, obtained by both relaxation and exact method of "Equivalent Elastic Panels."[6] The influence line is shown in figure (9).

7. Rise of Temperature in Bottom Chord.

This is equivalent to a relative axial translation δ similar to that in figure (8) in the bottom chords of all the panels in the Vierendeel; such that $\delta = \alpha ts$.

The computations shown in table 7 corresponding to these imposed translations are similar to those in the previous problem, table 5.

The Vierendeel is symmetrical, and the computations correspond to the

6. Thesis for Ph.D. London University 1948 by A. F. Diwan.

Table (3). Influence Line For M_{cd}
Moments produced due to unit relative rotation in CD.

	A			B			C			D	
1								11920	5960		
2								-6810	-6580		
3					-1072		382	135	-1110	344	
4					808	733		246	237		- 578
5		53		- 134	- 314	54.5	- 57	240	- 323	- 48	96.5
6		60	48.5		149	409		179	172		-283.5
7	28.5	- 10	- 30	-27.5	- 173	- 10	10	84	- 178	8.5	31
8		62.5	50.5		216	197		73	70		- 168
	9.0	- 7.0	- 215	-21.5	-74.5	- 7.5	- 3.5	51.5	-76.5	- 4	29
		39.5	52*		103	93.5		24	23		- 95
	7.5	- 3.5	- 11	- 8.5	- 33	- 3.5	1.0	27	- 34	0.5	17
		19.5	16		43	39		2.5	2		-58.5
	3.5	- 1.3	- 6.5	- 4.5	- 14	- 1.3	0.3	18	- 14	0.4	11.5
		9.8	8.0		18	16.3		- 8	- 7		-38.5
15	-127.5	-143.5	61.5	29	63.5	-3390	-1175	-3470	1125	338	1130

1								0.00	0.00		
2								-2290	-2200		
3					382		- 373	378	382	40.5	
4					275	254		82.5	78.5		- 192
5		- 102		25.0	- 57	- 102	- 110	- 53	- 57	72	- 32
6		21.5	17.0		155	140		60	57		-94.5
7	-26.5	- 21	11.5	- 4.7	10	- 21	- 60	9.5	10	25	24.5
8		22	18		74	68		-24.5	23.5		- 56
	- 19	- 16.5	3.5	- 3.4	- 3.5	-16.5	- 26	- 4.5	- 3.5	15.5	8.5
		14	11.5		35	32		8	7.5		-31.5
	- 10	- 6.5	3.0	- 1.6	1.0	- 6.5	-11.5	0.6	1.0	8.0	7.5
		7	5.5		15	13.5		1.0	0.5		-19.5
	- 5.5	- 3.5	1.4	- 0.6	0.3	- 3.5	- 5.0	0.4	0.3	5.8	4.0
		3.5	2.8		6.2	5.5		- 2.5	-2.5		-13.0
15	1 01	415	-297	- 388	- 297	669	669	669	664	602	664
	J			K			L			M	

All moments are multiplied by $1000/EI_o$

left half of the Vierendeel only. The displacement α ts taken in the computations equals $\frac{(10)^4}{EI_o}$.

The final moments are given in table (8), and are shown in fig (10).

CONCLUSION

The method just outlined is applicable to the case when the cross section of the members vary between the joints. The same type of special panel displacement is adopted, but both carry-over factors and balancing coefficients for the panel moments will be different, yet, they can be easily evaluated.

The rest of the computations will be unchanged.

	E			F			G			H	1
											2
135											3
- 577											4
241	- 29		96								5
- 284		- 70	- 73								6
84	22	- 5	31	9.5	-	-5.0					7
- 168		- 65	-67.5		-4.5	-4.5					8
51.5	8	7	28.6	0.2	2.0	6.5	-1.6		1.9		
- 95		-48.5	-50.5		-8.0	-9.0		-0.7	-0.9		
27	7	+ 6	16.5	1.6	1.0	6.0	0.4	-0.3	1.0	0.5	
-58.5		- 34	-35.5		-9.0	- 10		-1.0	-1.2		
18	4	5.5	11.5	0.9	1.1	5.5	0.2	0	1.0	0.2	
-38.5		-23.5	-24.5		-7.8	-8.5		-1.0	-1.2		
384	114	382	34.8	11.7	34	9.5	4,3	9.2	-0.7	-0.6	15

											1
											2
378											3
- 192											4
- 53	29		- 32								5
- 945		-23.5	-24.5								6
9.5	9	9.5	24.5	- 1.7		9.5					7
- 56		-21.5	-22.5		-1.5	-1.5					8
- 4.5	8.5	0.2	8.5	2.3	-1.2	0.2	0.9		-1.2		
-31.5		-16.0	-16.5	-	-2.8	-3.0		-0.2	-0.3		
0.6	5.0	1.6	7.5	2.0	0.3	1.6	0.5	0.2	0.3	-0.2	
-19.5		-11.5	- 12		-3.1	-3.4		-0.4	-0.5		
0.4	3.5	0.9	4.0	1.9	0.2	0.9	0.5	0	0.2	0	
-12.8		- 8.0	- 8.3		-2.7	-2.8		-0.4	-0.5		
26.3	24	26.3	25.8	25.8	25.8	-0.8	-1.00	-0.8	0.6	1.6	15
	N			O			P			R	

Table 4 : Chord moments due to a Relative Rotation $\phi = \frac{1000}{EI_o}$ in CD (I.L. for M^{cd})

	Relaxation method.	Exact method		Relaxation method	Exact method		Relax. method	Exact method		Relax. method	Exact method.
AB	79	78	ED	-282	-290	JK	- 40	- 39.3	NM	-36.7	-56
BA	147.5	147	EF	156	149	KJ	- 224	- 223	NO	-42.3	-42.3
BC	19.5	18	FE	- 32.7	- 42	KL	596	603	ON	-45.2	-51
CB	-1870	-1860	FG	9.0	10	LK	1032	1038	OP	15.0	13.0
CD	2712	2700	GF	- 9.5	- 14	LM	-1116	-1116	PO	0.7	- 3.0
DC	- 734	- 730	GH	6.2	7	ML	-1038	-1032	PR	- 1.6	- 1.2
DE	94	87	HG	- 0.1	- 1.0	MN	270	260	RP	- 1.4	- 1.9

	A			B			C			D	
1								9400	-9400		
2					-1985		- 523	2050	-2030	490	
3					2940	2670		- 210	- 202		-2925
4		-190	-	-96.5	- 845	- 196	- 116	1015	- 860	148	188
5		323	260		1370	1245		- 320	- 309		-1627
6	-14.7	-136	- 35	- 70	- 344	- 142	- 20	535	- 352	60	+ 180
7		234	190		572	520		- 320	- 309		- 961
	0.9	- 62	- 22	- 34	- 121	- 65	6.0	300	- 124	33	140
		114	92		205	186		- 272	- 262		- 591
	1.4	- 24	- 14	- 16	- 33	- 25	10	175	- 34	21	102
		49	40		53	48		- 208	- 200		- 376
	2.5	- 6	- 7	- 6	0	- 6	9	106	0	15	71
		16	13		- 3	- 3		- 150	- 144		- 243
	1.7	0.5	- 3	- 2	9	0.5	6.5	65	9	10	48
		2.5	2		- 22	- 20		- 105	- 102		- 158
	1.3	3.0	-1.3	0	11	3	5	42	11.5	7	33
		-2.4	- 2		- 22	- 20		- 75	-72.5		- 104
	0.7	3	-0.5	1	9.5	3	3.5	27	10	5	23
		- 4	- 3		-18.5	- 17		- 51	- 49		- 69
20	-14.7	-166	-823	- 388	- 846	-6576	-2286	-6724	8671	2591	8709

1					-			3140	-3140		
2					- 523		- 690	540	- 523	612	
3					1010	920		- 70	- 68		- 975
4		-73.5	-	- 89	- 117	-73.5	- 292	163	- 117	303	75
5		115	92.5		470	430		- 107	- 104		- 543
6	- 315	- 54	- 6	- 65	- 20	- 54	- 119	66	- 20	160	72
7		83	67		187	180		- 107	- 104		- 320
	- 19	- 26	0.5	- 29	6	- 26	- 43	36	6	90	51
		41	32		70	64	- [illegible]	- 91	- 87		- 197
	-12.5	- 12	0.6	-11.5	10	- 12	- 11	23	10	52	34
		17.5	14		18	17		- 70	- 67		- 126
	- 6.2	- 4.7	1.0	- 3	9	- 5	0	17	9	32	22.5
		6	5		- 1	- 1		- 50	- 48		-81.5
	- 3.0	- 1.6	0.8	0.3	6.5	- 1.5	3	11.5	6.5	19.5	14.5
		1.0	0.7		- 7.5	- 7		- 35	- 34		-52.5
	- 1.1	0	0.5	1.5	5	0	4	8	5	12.5	9
		- 0.9	-0.7		- 7.5	- 7		- 25	- 24		-34.5
	- 0.4	0.9	0.2	1.2	3	1	3	5.5	3	8	6
		- 1.5	-1.2		- 6.5	- 6		- 17	- 16		- 23
20	-11.3	- 4.5	-340	- 445	- 340	-1238	-1236	-1238	1753	1586	1753
	J			K			L			M	

Table (5) Influence Line for Thrust in LM.
(Moments produced by a unit relative axial translation in LM).
(All moments are multiplied by $10^4/EI_0$).

	E			F			G			H	
											1
2060											2
-2925											3
1020	68		188								4
-1627		- 290	-302								5
536	66	25	180	4.5		24					6
- 961		- 309	- 320		- 30	- 33					7
300	47	36	140	6.5	2	35	1.1	—	2.0		
- 591		- 254	- 274		- 47	- 51		- 3.1	-3.8		
+ 176	31	37	102	6.7	3.7	36	2.2	0.5	3.6	0.1	
- 376		- 194	- 202		-49.7	- 55		- 5.5	-6.5		
106	20	30	70.5	6.0	4.5	29.5	2.4	0.8	4.4	0.1	
--243		- 139	- 144		- 4[illegible]	- 47		- 6.3	-7.8		
66	13	23	48	4.8	4	22.5	1.6	0.9	4.0	0.1	
- 158		- 97	- 100		- 33	- 36		- 5.4	-6.6		
42	20	16	33	3.5	3.5	16	1.5	0.7	3.3	0	
- 104		- 66	- 69		- 24	- 26		- 4.5	-5.7		
27	5.5	11	22.5	2.5	2.5	11	1.0	0.5	2.5	0.1	
- 69		- 45	-46.5		- 17	- 18.3		- 3.5	-4.0		
1601	478	1600	372	128	364	44.5	20	43.4	7.6	6.6	20

											1
540											2
- 975											3
163	56		75								4
- 543		- 97	--100								5
66	54	4	72	8.2		4					6
- 320	-	- 103	- 108		- 10	- 11					7
36	42	6	51	12	1.0	6.5	0.9		1.1		
- 197		- 85	- 8[illegible]		- 1[illegible]	-17.5		-1.1	-1.4		
23	30	7	34	12.5	1.7	6.7	1.7	0	1.7	0.4	
- 126		- 65	- 67		- 17	-18.5		-1.9	-2.3		
17	21	6	22.5	10	2	6	2.0	0	1.8	0.7	
-81.5		-46.5	- 48		-14.5	- 15		-2.1	-2.8		
11.5	14.5	5	14.5	8	1.5	5	2.0	0	1.4	0.8	
-52.5		- 32	-33.5		-11.5	-12.5		-1.9	-2.4		
8	10	3.5	9	5.5	1.0	3.5	1.5	0	1.0	0.6	
-34.5		- 22	- 23		- 8	- 9		-1.6	-2.0		
5.5	6.5	2.5	6	4	1.0	2.5	1.1	0	0.8	0.5	
- 23		- 15	-15.5		- 6	- 6.5		-1.0	-1.5		
576	526	576	71	72	71	17	22.2	17	0.5	1.1	20
	N			O			P			R	

Table (5) (Cont.)

Table (6) Moments Produced by a Relative Axial Translation $\delta = \frac{10^4}{EI_o}$ in L M.

	Relaxation Method	Exact Method		Relaxation Method	Exact Method		Relax. Method	Exact Method		Relaxation Method	Exact Method.
AB	153	151	ED	-1120	-1149	JK	84.5	84	NM	- 907	- 916
BA	- 314	- 313	EF	384	384	KJ	- 132	132	NO	146	132
BC	925	918	FE	- 301	- 317	KL	782	771	ON	- 128	- 142
CB	-2395	-2376	FG	139	1210	LK	181	156	OP	- 4	- 4
CD	5300	5250	GF	-48.5	- 57	LM	2200	2205	PO	-38.8	- 45
DC	-5820	-5760	GH	18.5	20	ML	-2560	2565	PQ	7.4	7.1
DE	2440	2376	HG	- 7	- 9	MN	- 315	-316	QP	- 4:1	- 5.4

Table 7. Moments Broduce by a Relative Rise of temperature in Bottom chard ($\alpha ts = \frac{10^4}{EI_o}$)

	A			B			C			D	
1		16200	-16200		11000	-11000		9400	-9400		8940
2	-2052	1040	- 4290	388	342	1070	107	95	350	26	- 95
3		-1840	- 1490		- 5900	- 5350		-2460	-2360		-
4	1485	1990	775	545	1380	2050	374	430	1410	108	- 430
5		- 2240	- 1810		- 2860	- 2600		-1830	-1760		-
	- 98	394	- 330	-43	332	406	84	146	340	30	- 146
		- 402	- 326		- 1130	- 1030		- 770	- 740		-
	166	300	28	54	245	308	85	112	250	24	- 112
		- 325	- 265		- 590	- 532		- 418	- 400		-
	17	105	- 37	0	94	108	28	52	96	6	- 52
		- 108	- 88		- 264	- 238		- 193	- 186		-
	25	59	- 4	7	51	61	18	29	52	4	- 29
		- 60	- 50		- 130	- 117		- 76	- 73		-
14	-6883	- 7773	8210	3880	8470	4924	1710	5043	1800	537	1803

1		5750	- 5750		3790	- 3790		3140	-3140		2980
2	-3810	296	- 822	468	107	296	120	28	107	28	- 28
3		- 655	- 528		-2000	- 1850		- 840	- 800		-
4	670	406	600	935	374	406	478	120	374	135	- 120
5		- 800	- 645		- 980	- 900		- 610	- 580		-
	- 290	- 43	- 40	185	84	- 43	116	33	84	43	- 33
		- 143	- 115		- 390	- 354		- 256	- 246		-
	25	42	68	140	85	42	85	26	85	34	- 26
		- 116	- 95		- 202	- 180		- 139	- 132		-
	- 33	0	7	60	28	0	33	7	28	16	- 7
		- 38	- 31		- 90	- 63		- 65	- 62		-
	- 3	5	10	28	18	5	18	4	18	9	- 4
		- 22	- 18		- 45	- 41		- 25	- 24		-
14	- 875	- 35 5	1430	1894	1430	1376	1376	1376	436	395	436
	J			K			L			M	

Table (8): Moments Produced by a Rise of Temperature

$\alpha ts = \dfrac{10^4}{EI_0}$ in Bottom Chord

	Relaxation method	Exact method		Relaxation method	Exact method		Relaxation method	Exact method
AB	+7340	7340	DC	-10615	-10250	LK	-5030	-5200
BA	-15870	-16060	DE	9880	9800	LM	2804	2940
BC	11040	11100	JK	4315	4270	ML	-3857	-3800
CB	-11940	-12130	KJ	-5930	-6030	MN	3197	3200
CD	9535	9800	KL	2210	2080	-	-	-

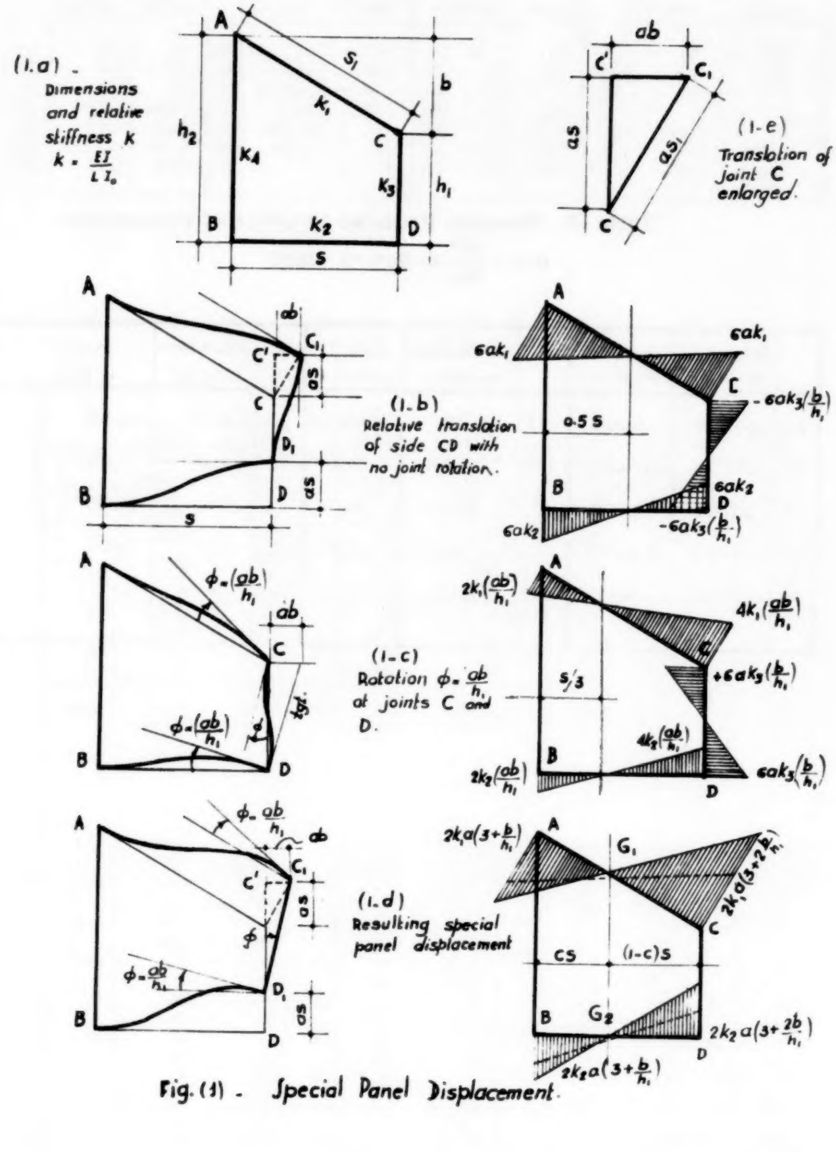
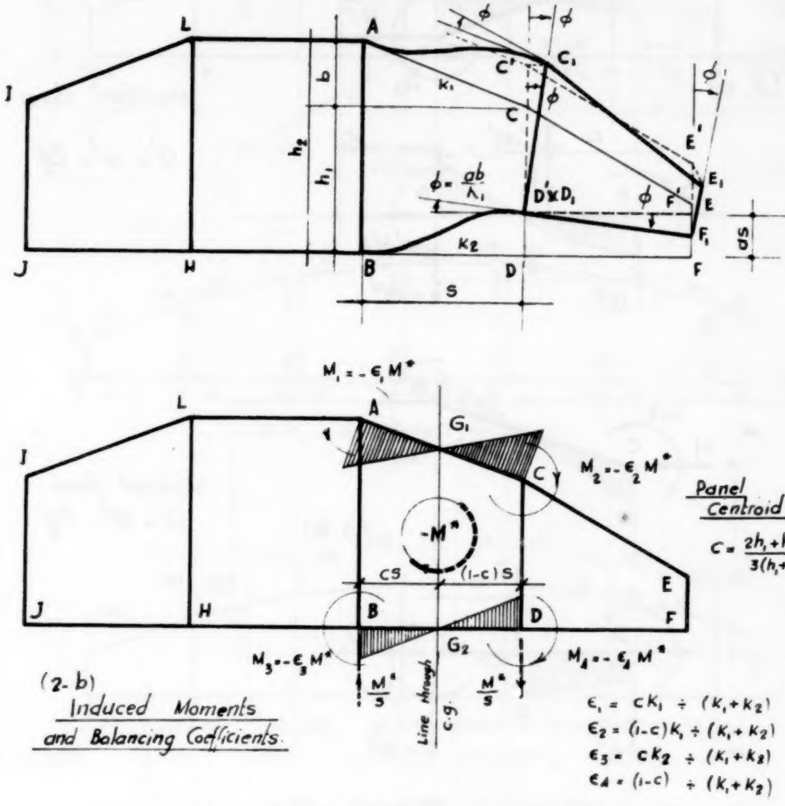
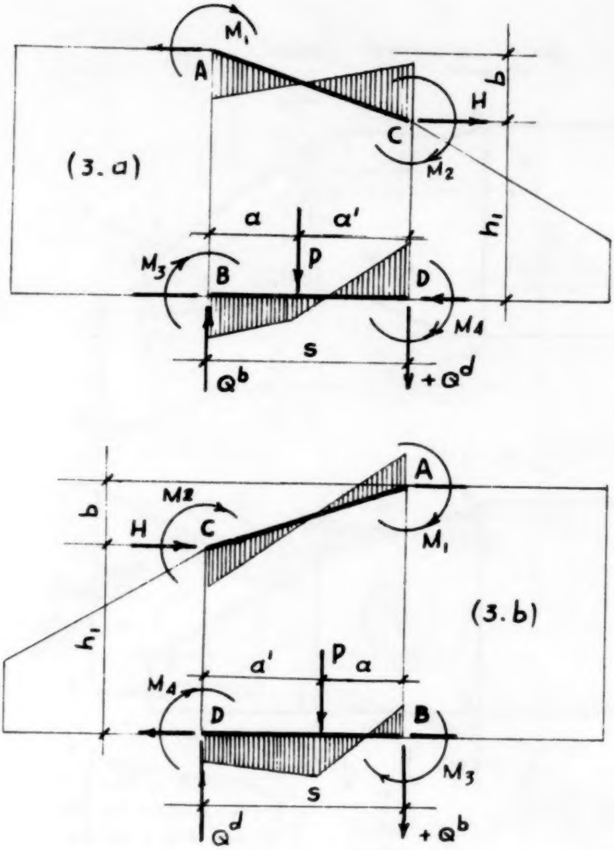


Fig. (1) - Special Panel Displacement.

(2. a) - Special Displacement Imposed on panel ABCD.



(Fig. 2) Special Panel Displacement and Balancing Coefficients ϵ.



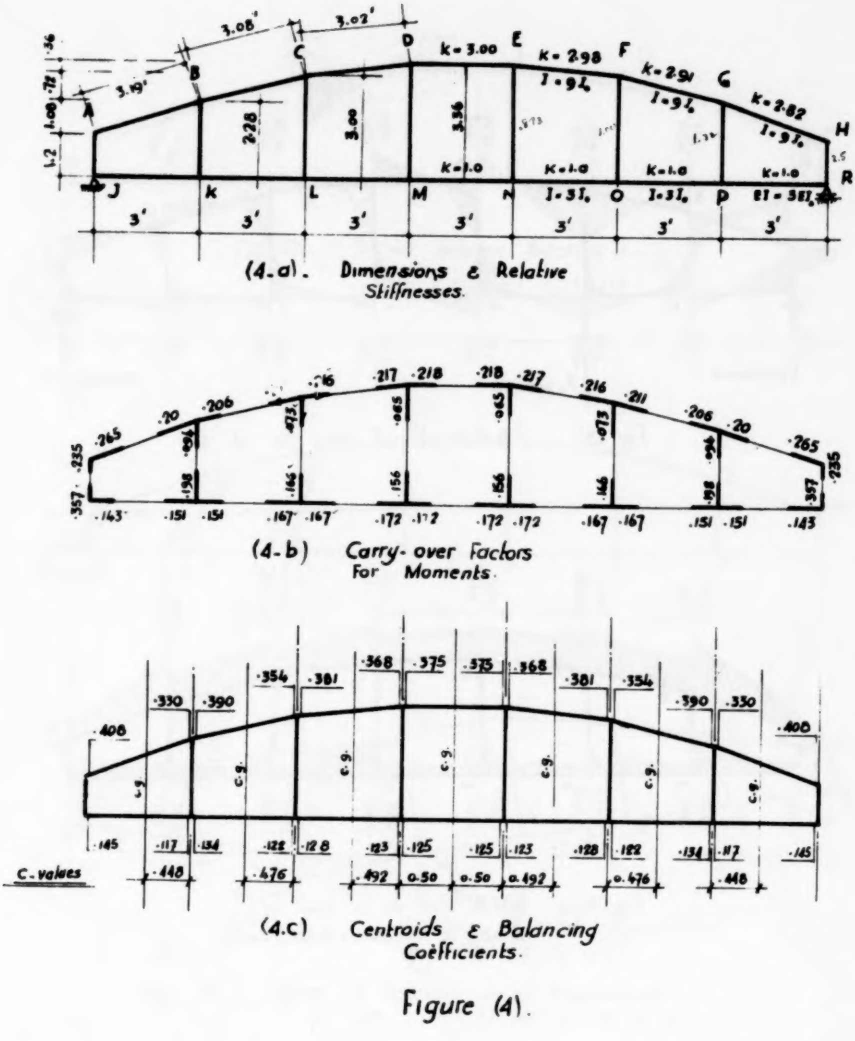
Modified Shear

$$Q' = Q^d + \frac{Pa}{s}$$

Modified Shear.

$$Q' = Q^d - \frac{Pa}{s}$$

Fig. (3). Static Equilibrium of Panel ABCD.



(4-a). Dimensions & Relative Stiffnesses.

(4-b) Carry-over Factors For Moments.

(4.c) Centroids & Balancing Coefficients.

Figure (4).

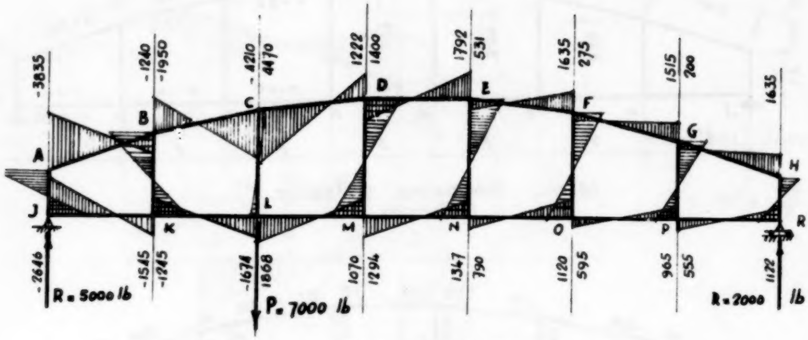


Fig. (5) B.M.D for P = 7000 lb at L

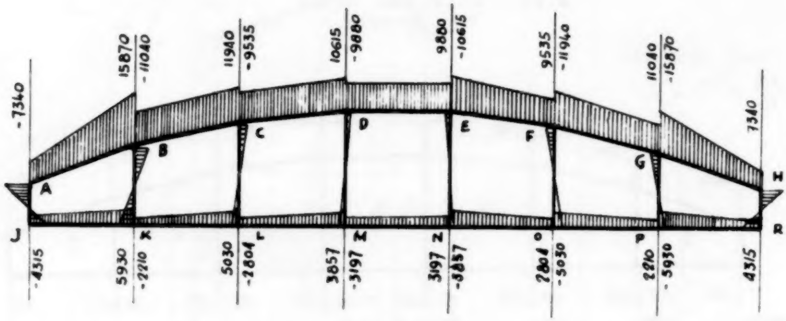


Fig. 10. B.M.D due to a Drop of Temp. in bottom chord. $\alpha t's = \frac{10^6}{EI_0}$

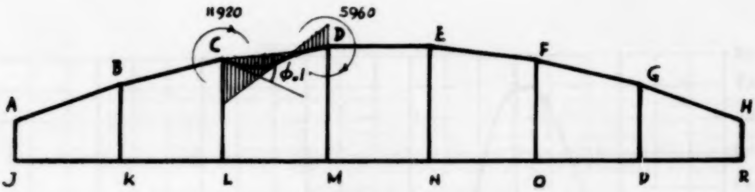
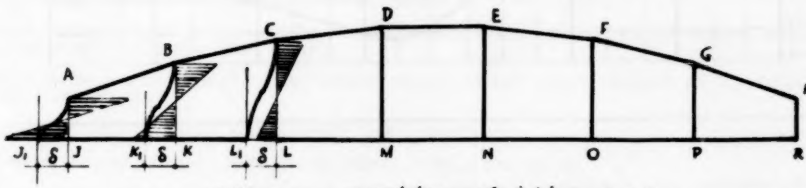
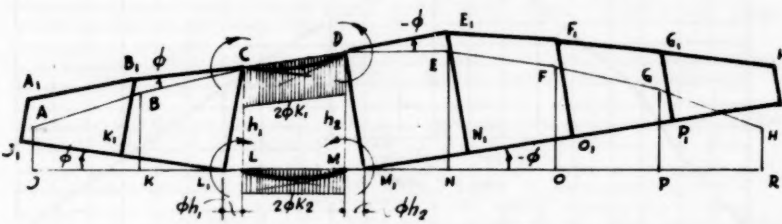


Fig. (6). Initial Relative Rotation $\phi = \frac{10^3}{EI_0}$ in CD; (For I.L. for M^{cd}).



TYPE a. Translation of Joints L, K & J without any rotation.



TYPE b. Rotations ϕ & $-\phi$ about C & D (L & M.)

Fig. (8) Types of Relative Axial Translation in LM. (For I.L for thrust in LM.)

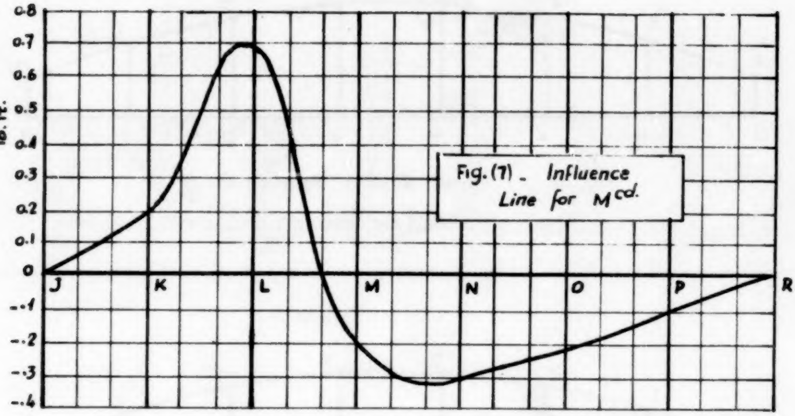


Fig. (7). Influence Line for $M^{cd.}$

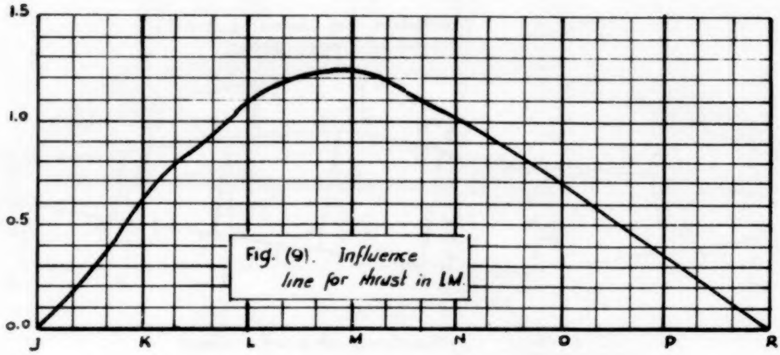


Fig. (9). Influence line for thrust in LM.

PROCEEDINGS-SEPARATES

The technical papers published in the past year are presented below. Technical-division sponsorship is indicated by an abbreviation at the end of each Separate Number, the symbols referring to: Air Transport (AT), City Planning (CP), Construction (CO), Engineering Mechanics (EM), Highway (HW), Hydraulics (HY), Irrigation and Drainage (IR), Power (PO), Sanitary Engineering (SA), Soil Mechanics and Foundations (SM), Structural (ST), Surveying and Mapping (SU), and Waterways (WW) divisions. For titles and order coupons, refer to the appropriate issue of "Civil Engineering" or write for a cumulative price list.

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DECEMBER: 359(AT), 360(SM), 361(HY), 362(HY), 363(SM), 364(HY), 365(HY), 366(HY), 367(SU)[c], 368(WW)[c], 369(IR), 370(AT)[c], 371(SM)[c], 372(CO)[c], 373(ST)[c], 374(EM)[c], 375(EM), 376(EM), 377(SA)[c], 378(PO)[c].

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JANUARY: 379(SM)[c], 380(HY), 381(HY), 382(HY), 383(HY), 384(HY)[c], 385(SM), 386(SM), 387(EM), 388(SA), 389(SU)[c], 390(HY), 391(IR)[c], 392(SA), 393(SU), 394(AT), 395(SA)[c], 396(EM)[c], 397(ST)[c].

FEBRUARY: 398(IR)[d], 399(SA)[d], 400(CO)[d], 401(SM)[c], 402(AT)[d], 403(AT)[d], 404(IR)[d], 405(PO)[d], 406(AT)[d], 407(SU)[d], 408(SU)[d], 409(WW)[d], 410(AT)[d], 411(SA)[d], 412(PO)[d], 413(HY)[d].

MARCH: 414(WW)[d], 415(SU)[d], 416(SM)[d], 417(SM)[d], 418(AT)[d], 419(SA)[d], 420(SA)[d], 421(AT)[d], 422(SA)[d], 423(CP)[d], 424(AT)[d], 425(SM)[d], 426(IR)[d], 427(WW)[d].

APRIL: 428(HY)[c], 429(EM)[c], 430(ST), 431(HY), 432(HY), 433(HY), 434(ST).

MAY: 435(SM), 436(CP)[c], 437(HY)[c], 438(HY), 439(HY), 440(ST), 441(ST), 442(SA), 443(SA).

JUNE: 444(SM)[e], 445(SM)[e], 446(ST)[e], 447(ST)[e], 448(ST)[e], 449(ST)[e], 450(ST)[e], 451(ST)[e], 452(SA)[e], 453(SA)[e], 454(SA)[e], 455(SA)[e], 456(SM)[e].

JULY: 457(AT), 458(AT), 459(AT)[c], 460(IR), 461(IR), 462(IR), 463(IR)[c], 464(PO), 465(PO)[c].

AUGUST: 466(HY), 467(HY), 468(ST), 469(ST), 470(ST), 471(SA), 472(SA), 473(SA), 474(SA), 475(SM), 476(SM), 477(SM), 478(SM)[c], 479(HY)[c], 480(ST)[c], 481(SA)[c], 482(HY), 483(HY).

SEPTEMBER: 484(ST), 485(ST), 486(ST), 487(CP)[c], 488(ST)[c], 489(HY), 490(HY), 491(HY)[c], 492(SA), 493(SA), 494(SA), 495(SA), 496(SA), 497(SA), 498(SA), 499(HW), 500(HW), 501(HW)[c], 502(WW), 503(WW), 504(WW)[c], 505(CO), 506(CO)[c], 507(CP), 508(CP), 509(CP), 510(CP), 511(CP).

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NOVEMBER: 534(HY), 535(HY), 536(HY), 537(HY), 538(HY)[c], 539(ST), 540(ST), 541(ST), 542(ST), 543(ST), 544(ST), 545(SA), 546(SA), 547(SA), 548(SM), 549(SM), 550(SM), 551(SM), 552(SA), 553(SM)[c], 554(SA), 555(SA), 556(SA), 557(SA).

DECEMBER: 558(ST), 559(ST), 560(ST), 561(ST), 562(ST), 563(ST)[c], 564(HY), 565(HY), 566(HY), 567(HY), 568(HY)[c], 569(SM), 570(SM), 571(SM), 572(SM)[c], 573(SM)[c], 574(SU), 575(SU), 576(SU), 577(SU), 578(HY), 579(ST), 580(SU), 581(SU), 582(Index).

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c. Discussion of several papers, grouped by Divisions.
d. Presented at the Atlanta (Ga.) Convention of the Society in February, 1954.
e. Presented at the Atlantic City (N.J.) Convention in June, 1954.

AMERICAN SOCIETY OF CIVIL ENGINEERS